Prednáška č.1

ADM

1. Priebezny test – 20 bodov  
   v 6.tyzdni z uciva 1. Az 5. Tyzdna
2. Priebezny test – 20

V 11. Tyzdni z uciva 6. Az 10. Tyzdna

1. Skuska – 60 b

Podmienka je aspon z oboch priebeznych 20b

Priebezne testy vo forme vybdru z viacnasobnych odpovedi

Opravny test nieje

45min az 1hod, bude sa konat pravdepodobne pred prednaskou

**Lineárna algebra**

Pracuje s lineárnymi vzťahmi, systémy lineárnych rovníc a ich riešenia

Lineárny vzťach dvoch premenných x a y:

a1x+a2y = b; a1,a2,b € R

o n premenných

a1x1+y2x2+…+anxn = b

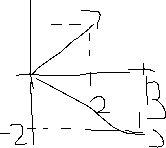
priklad linearnych vztahov

x+9y = 29, y = 2/5x -1

**Vektory v dvojrozmernom priestore**

Vektor v rovine je usporiadaná dvojica u-> = (u1, u2), kde u1,u2 € R

Pr.: u-> = (2,1)



Vector udava smer

u->+v-> = (2,1)+(1,-3) = (3,-2)

3u->=(6,3)

**Parametrické vyjadrenie priamky**

X->=A-> + €u->, t€R -> x = a1 + t u1 ; y = a2 + t u2

x-> = (x,y) A = (a1, a2)



Všeob. ax + by = c



**Vektor v 3D priestore** je u-> = (u1,u2,u3)



au-> = (au1,au2,au3) => zmena velkosti a-krát

Parametrické vyjadrenie roviny

X->=A->+su->+tv-> s,t€R (s a t su parametre)

Všeobecne

Ax + by +cz = d => n-> = (a,b,c) n-> je kolmy na u->, v->

A,b,c,d € R

Všeobecne sa to zapisuje ako prienik 2 rovín

a1x+b1y+c1z=d1

a2x+b2x+c2z = d2

**Lineárna kombinácia vektorov:**

n-rozmerny vector v-> je linearnou kombinaciou n-rozmernych vektorov v1->,v2->,…,vk-> ak plati

v->=c1v1->+c2v2->+…+ckvk->

kde c1,c2,…,ck €R

Príklad: Vektor (2,1,5,-5) je lineárnou kombinaciou vektorov

(1,2,1,-1),(1,0,2,-3),(1,1,0,-2, lebo (2,1,5,-5) = 1\*(1,2,1,-1)+2\*(1,0,2,-3)-1\*(1,1,0,-2)

Priklad: Zistite ci je vektor (8,9,11) linearnou kombinaiou vektorov (1,3,4),(-2,-1,-5),(1,-2,-1)

(8,9,11)=c1(1,3,4)+c2(-2,-1,-5)+c3,(1,-2,-1)

8 = c1-2c2+c3

9 = 3c1-c2-2c3 \*\*

11=4c1-5c2-c2

**Lineárna závislosť vektorov:** množina n-rozmerných vektorov S = {v1->,v2->,…,vk->} je lineárne závisla ak existuju konštantny c1,c2,…,ck, ktore su nie vsetky nulove take, ze

C1v1->+c2v2->+…+ckvk-> = (0,0,…,0)

V opačnom prípade sa mnozina S nazyva linearne nezavislou; t.j. ak c1=c2=…=ck = 0

Priklad: mnozina vektorov {(1,2,-1),(1,-2,1),(-3,2,-1),(2,0,0)} je linearne zavisla pretoze

1(1,2,-1)+1(1,-2,1)+0(-3,2,-1)-1(2,0,0) = (0,0,0);

Priklad: su vektory /3,3,4),(1,5,4),(2,4,3) linearne nezavisle?

C1(3,3,4)+c2(1,5,4)+c3(2,4,3) = (0,0,0)

3c1+c2+2c3 = 0

3c1,5c2+4c3 = 0 \*\*

4c1+4c2+3c3 = 0

**Počet riešení sústavy lineárnych rovníc v R**

Riešením sústavy n linearnych rovnic je usporiadana n-tica realnych cisiel, ktora vyhovuje kazdej rovnici danej sustavy.

Pocet rieseni kazdej sustavy linearnych rovnic v R je jedna z nasledujucich moznosti:

1. Ziadne riesenie



1. Jedine riesenie



1. Nekonecne vela rieseni



**Riešenie sústavy lineárnych rovníc**

**Elementárne operácie,** ktoré nemenia množinu riešení sústavy rovníc:

ERO 1 - Výmena poradia l’ubovol’ných dvoch riadkov.

ERO 2 - Vynásobenie riadku nenulovou konštantou.

ERO 3 - Pripočítanie nenulového násobku jedného riadku k inému

riadku.

x-2y+z=8

3x-y-2z=9

4x-5y-z=11

ERO3: x-2y+z=8

R2-3R1 5y-5z=-15

R3-4R1 3y-5z=-21

R2\*1/5 y-z=-3

(delime 5)

x-2y+z=8

y-z=-3

R3-3R2 -2z = -12

R1 x-2y+z=8

y-z=-3

R3: (-2) z = 6

R1-R3 x-2y = 2 R1+2R2 x= 8

X=8;y=3,z=6

|  |  |  |  |
| --- | --- | --- | --- |
| 1 | -2 | 1 | |8 |
| 3 | -1 | -2 | |9 |
| 4 | -5 | -1 | |11 |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1 | -2 | 1 | |8 |  |
| 0 | 5 | -5 | |-15 |  |
| 0 | 3 | -5 | |-21 |  |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1 | -2 | 1 | |8 |  |
| 0 | 1 | -1 | |-3 |  |
| 0 | 0 | -2 | |-12 |  |

|  |  |  |  |
| --- | --- | --- | --- |
| 1 | -2 | 1 | |8 |
| 0 | 1 | -1 | |-3 |
| 0 | 0 | 1 | |6 |

|  |  |  |  |
| --- | --- | --- | --- |
| 1 | -2 | 0 | |2 |
| 0 | 1 | 0 | |2 |
| pica |  |  | | |

3x+2y-3z = 3

4x+6y+z = -1

7x + 4y -4z = 12

|  |  |  |  |
| --- | --- | --- | --- |
| 3 | 2 | -3 | |3 |
| 4 | 6 | 1 | |-1 |
| 7 | 4 | -4 | |12 |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1 | 4 | 4 | |-4 | R2-R1 |
| 4 | 6 | 1 | |-1 |  |
| 7 | 4 | -4 | |12 |  |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1 | 4 | 4 | |-4 |  |
| 0 | -10 | -15 | |15 | R2-4R1 |
| 0 | -24 | -32 | |40 | R3-7R1 |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1 | 4 | 4 | |-4 |  |
| 0 | 1 | 1,5 | |-1,5 | R2/10 |
| 0 | 3 | 4 | |-5 | R3/-8 |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1 | 0 | -2 | |2 | R1-4R2 |
| 0 | 1 | 1,5 | |-1,5 |  |
| 0 | 0 | -0,5 | |-0,5 | R3-3R2 |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1 | 0 | -2 | |2 |  |
| 0 | 1 | 1,5 | |-1,5 |  |
| 0 | 0 | 1 | |1 | R3/-2 |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1 | 0 | 0 | |4 | R1+2R3 |
| 0 | 1 | 0 | |-3 | R2-1,5R3 |
| 0 | 0 | 1 | |1 |  |

X=4

Y=-3

Z=1

Skuska plati

Príklad č.3

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1 | 1 | 2 | 1 | 2 |
| 2 | -1 | 1 | -1 | 7 |
| 4 | -2 | 1 | 1 | -6 |
| -1 | 4 | 3 | 1 | 20 |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| 1 | 1 | 2 | 1 | 2 |  |
| 0 | -3 | -3 | -3 | 3 | R2-2R1 |
| 0 | -6 | -7 | -3 | -14 | R3-4R1 |
| 0 | 5 | 5 | 2 | 22 | R4+R1 |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| 1 | 1 | 2 | 1 | 2 |  |
| 0 | 1 | 1 | 1 | -1 | R2/(-3) |
| 0 | -6 | -7 | -3 | -14 |  |
| 0 | 5 | 5 | 2 | 22 |  |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| 1 | 0 | 1 | 0 | 3 | R1-R2 |
| 0 | 1 | 1 | 1 | -1 |  |
| 0 | 0 | -1 | 3 | -20 | R3+6R2 |
| 0 | 0 | 0 | -3 | 27 | R4-5R2 |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| 1 | 0 | 0 | 3 | -17 | R1+R3 |
| 0 | 1 | 0 | 4 | -21 | R2+R3 |
| 0 | 0 | 1 | -3 | 20 | -R3 |
| 0 | 0 | 0 | 1 | -9 | R4/-3 |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| 1 | 0 | 0 | 0 | 10 | R1-3R4 |
| 0 | 1 | 0 | 0 | 15 | R2-4R4 |
| 0 | 0 | 1 | 0 | -7 |  |
| 0 | 0 | 0 | 1 | -9 |  |

X=10

Y=15

Z=-7

W=-9

Priklad c.4

6x+3y+2z=6

7x+2y-z=9

5x+4y+5z=-4

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 6 | 3 | 2 | 6 |  |
| 7 | 2 | -1 | 9 |  |
| 5 | 4 | 5 | -4 |  |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1 | -1 | -3 | 10 | R1-R3 |
| 7 | 2 | -1 | 9 |  |
| 5 | 4 | 5 | -4 |  |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1 | -1 | -3 | 10 |  |
| 0 | 9 | 20 | -61 | R2-7R1 |
| 0 | 9 | 20 | -54 | R3-5R1 |
| 1 | -1 | -3 | 10 |  |
| 0 | 9 | 20 | -61 |  |
| 0 | 0 | 0 | 7 | R3-R2 |

Nema riesenia lebo 0+0+0 sa nevie rovnat nejakemu cislu (v tomto pripade 7)

X + y + z =7

-2x + 4y + z = -5

5x + 11y + 8z = 44

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1 | 1 | 1 | 7 |  |
| -2 | 4 | 1 | -5 |  |
| 5 | 11 | 8 | 44 |  |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1 | 1 | 1 | 7 |  |
| 0 | 6 | 3 | 9 | R2+2R1 |
| 0 | 6 | 3 | 9 | R3-5R1 |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1 | 1 | 1 | 7 |  |
| 0 | 1 | ½ | 3/2 | R2/6 |
| 0 | 0 | 0 | 0 | R3-r2 |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1 | 0 | 0,5 | 5,5 |  |
| 0 | 1 | 0,5 | 1,5 |  |
| 0 | 0 | 0 | 0 |  |

X+0,5z = 5,5

Y + 0,5z = 1,5 z = t; t € R (parameter)

Riešenie: x = 5,5 - 0,5t; y = 1,5 – 0,5t; z = t; t € R

5,5-0,5t+1,5-0,5t+t=7 => 7 = 7 /\*-2

-11+t+6-2t+t=-5 => -5 = -5

27,5-2,5t+16,5-5,5t+8t=44 => 44 = 44

**Gaussova eliminačná metóda – neformálny opis**

Etapa 1: Identifikujeme pivotne prvky (prve nenulove prvky v radnu) a pomocou ero 2 z nich produkujeme pivotne jednotky pocnuc lavym hornym prvkom(jeho ziskanie mozeme vyzadovat pouzitie ERO1) a pokracujuc postupne v parvo a nadol. Ihned po ziskani pivotnej jednotky vyprodukujeme pomocou ERO3 nuly v stlpci pod nou. Pripadne nulove riadky umiestnime pod nenulovymi riadkami pomocou ERO1

Etapa 2: Pomocou ERO 3 postupne produkujeme nuly and pivotnymi jednotkami, pocnuc stlpcom s poslednou pivotnou jednotkou vpravo dolu a pokracujuc smerom vlavo a nahor.

**Redukovany tvar**

Matica v redukovanom tvare sa nazyva matica v ktorej:

Pivotny (prvy lavy nenulovy) prvok v kazdom riadku je rovny 1

Pivotna jednotka v kazdom riadku sa nachadza vpravo od pivotnych jednotiek vo vsetkych vyssie polozenych riadkoch

V kazdom stlpci s pivotnou jednotkou su vsetky ostatne prvky nulove

Pripadne riadky obsahujuce same nuly nasleduju az za vsetkymi riadkami obsahujucimi pivotne jednotky

**Redukovany tvar je pre kazdy system rovnic jednoznacne urceny.**

**Hodnost matice** je pocet linearne nezavislych riadkov v matici.

Je totozna poctu nenulovych riadkov zodpovedajucej matice v redukovanom tvare

Priklad c.6

X + 2y + 3z – 7v = 5

2x – y + z -4v = 5

X -2y-z+v = 1

-2x + 3y + z = -3

Vysledok (postup sprav doma)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1 | 0 | 1 | -3 | 3 |
| 0 | 1 | 1 | -2 | 1 |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |

Priklad c.8

3x+y+2z=0

3x+5y+4z=0 \*\*

4x+4y+3z=0

Vysledok je vzdy 0 (homogenna sustava lebo sa vsetko rovna 0)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 3 | 1 | 2 | 0 |  |
| 3 | 5 | 4 | 0 |  |
| 4 | 4 | 3 | 0 |  |

Skus doma

**Sustavy rovnic s inym poctom rovnic ako neznamych**

Ak ma linearna sustava rovnic viac rovnic ako neznamych, pocet rieseni bude jedna z moznosti

Ziadne

Jedine

Nekonecne vela

Ak ma linearna sustava rovnic menej rovnic ako neznamych

Ziadne

Nekonecne vela